(b) Calculate the Rank Correlation Coefficient of the following data :

x	52	53	42	60	45	41	37	38	25	27
y	65	68	43	38	77	48	35	30	25	50

9. A set of 5 coins is tossed 3200 times and the number of heads appearing each time is noted. The results are given below :

No. of heads	0	1	2	3	4	5
Frequency	80	570	1100	900	500	50

Test the hypothesis that coins are unbiased

Roll No. ....

3057

B. Tech (Mech. Engg.) 3rd Semester Examination – February, 2022

MATHEMATICS-III (PDE, Probability & Statistics)

Paper : BSC-ME-203-G

Time : Three hours] *[Maximum Marks : 75*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Question No. 1 is **compulsory**. Attempt total *five* Questions with selecting *one* question from each Unit. All questions carry equal marks.

1. (a) Write down one dimensional Heat and Wave equations. 2.5×6

(b) What are the assumptions of bivariate normal distribution ?

(c) If $P(A) = \frac{6}{11}$, $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$. Find $P(B/A)$.

(d) Define Poisson Distribution. What are applications of Poisson distribution ?

(e) Solve :

$$2r + 5s + 2t = 0$$

(f) Define χ^2 test as a goodness of fit.

SECTION – A

2. (a) Solve :

$$x(y - z)p + y(z - x)q - z(x - y) = 0$$

(b) Solve :

$$(D^2 - DD' - 6D'^2)z = 4\cos(2x + y)$$

3. (a) Find the general solution of the partial differential equation :

$$(D^2 - D' - 1)z = x^2 y$$

(b) Solve :

$$(x^2D^2 - y^2D'^2)z = x^2 y$$

SECTION – B

4. (a) Using method of separation of variables, $4\left(\frac{\partial u}{\partial x}\right) + \left(\frac{\partial u}{\partial y}\right) = 3u$, given $u = 3e^{-y} - e^{-5y}$ when $x = 0$.

(b) Solve the one dimensional diffusion equation $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}\frac{\partial u}{\partial t}$ in the range $0 \le x \le 2\pi$, $t \ge 0$ subject to the boundary conditions : $u(x, 0) = \sin^3 x$ for $0 \le x \le 2\pi$ and $u(0, t) = u(2\pi, t) = 0$ for $t \ge 0$.

5. Solve $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, which satisfies the boundary conditions $u(0, y) = u(a, y) = u(x, 0) = 0$ and $u(x, b) = \sin\left(\frac{n\pi x}{a}\right)$.

SECTION – C

6. (a) Let x be a random variable defined by the density function $f(x) = \begin{cases} 3x^2, & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

Find $E(x)$, $E(3x - 2)$, $E(x^2)$.

(b) State and Prove Baye's Theorem

7. (a) The mean and variance of a sample of 25 measurement are 75 and 100 respectively. Use Chebychev's inequality to describe the distribution of measurements.

(b) Out of a lot containing 5 good, 4 faulty and 3 partially faulty but working batteries, three have been selected at random with independent. Find the probability that selection consists of exactly one of each type.

SECTION – D

8. (a) How will you measure kurtosis of a distribution ? How does it differ from skewness ?